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Crack identification in plates using wavelet analysis

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Abstract

In this paper, a method for crack identification in plates based on wavelet analysis is presented. The case of an all-over part-through crack parallel to one edge of the plate is considered. The vibration modes of the plate are analyzed using the continuous wavelet transform and both the location and depth of the crack are estimated. The position of the crack is determined by the sudden change in the spatial variation of the transformed displacement response. To estimate the depth of the crack, an intensity factor is defined which relates the depth of the crack to the coefficients of the wavelet transform. An intensity factor law is established which allows accurate prediction of crack depth. The viability of the proposed approach is demonstrated using simulation examples. In view of the obtained results, the advantages and limitations of the proposed approach as well as suggestions for future work are presented and discussed.

1. Introduction

Because of its practical importance, crack identification in structures has been the subject of intensive investigations during recent decades. As a result, a plethora of analytical, numerical and experimental research work now exists. In this connection, vibration analysis has been proved a fast and inexpensive method for effective crack identification. A review of the state of the art of vibration-based methods for testing cracked structures has been published by Dimarogonas [1].

The existence of a crack in a structure results in a reduction of stiffness which in turn leads to a decrease in natural frequencies and changes in the mode shapes of vibration. An analysis of these changes makes it possible to identify cracks. Dimarogonas [2] and Anifantis et al. [3] modelled the crack as a local flexibility and computed the equivalent stiffness using fracture mechanics methods. In that vein, they developed methods to identify cracks in beams relating crack depth to the change in natural frequencies [4,5] Adams and Cawley [6] have developed an experimental technique to estimate the location and depth of a crack from changes in natural frequencies.

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Further research work on crack identification via natural frequency changes was done by Massoud et al. [7] and Narkis [8]. A variational approach to the problem of cracked beams has been used by Chondros et al. [9]. They developed a continuous vibration theory of cracked Bernoulli–Euler beams and they reported results for the variation of the fundamental frequency of a simply supported cracked beam. The methodology of crack detection based on natural frequency changes has been also closely followed in studies of multicracked beams, including those by Shen and Pierre [10], Ruotolo et al. [11], Sekhar [12] and Schifrin and Ruotolo [13].

The main reason for the popularity of natural frequencies as a damage indicator is that natural frequencies are rather easy to determine with a high degree of accuracy. Problems exist, however, when the size of the crack is small. The existing methods give a proper estimation of moderate cracks (about 20% of the height of the beam).

The idea of using mode shapes as a crack identification tool is the fact that the presence of a crack causes changes in derivatives of the mode shapes at the position of the crack. Rizos et al. [14], for example, suggested a method for using measured amplitudes of a cantilever beam vibrating at one of its natural modes to identify crack location and depth. To get accurate estimates of the mode shapes, however, one needs detailed measurements at the site of interest. This fact increases considerably the duration of the investigation and this is the main disadvantage of using mode shapes for crack identification.

Vibration analysis has been mainly concentrated on crack detection in beam structures. The vibrational behaviour of cracked plates has been also studied with no emphasis, however, on crack detection. Lee [15] used the Rayleigh–Ritz method to obtain fundamental frequencies of annular plates having internal concentric cracks. Lee and Lim [16] have investigated the vibrational behaviour of a rectangular plate with a centrally located crack. Finite element methods have also been used for vibration analysis of damaged rectangular plates [17,18]. Recently, Khadem and Rezaee [19] developed an analytical approach for crack detection in rectangular plates. The flexibility of the crack was modelled as a line spring with varying stiffness along the crack. The obtained results show that moderate cracks have a minor effect on the natural frequencies of the cracked plate making an accurate crack detection difficult. To improve the accuracy of their analysis, the authors used modified comparison functions for the prediction of natural frequencies in case of a plate with a crack of arbitrary length [20].

In the light of the above discussion, it is obvious that the literature on crack detection, both in beams and plates, has been so far dominated by studies based on methods that utilize natural frequency changes. Having in mind that small cracks have a minor effect on natural frequencies of a plate, more sensitive methods capable of detecting small changes become important. Recently, methods based on wavelet analysis are emerging and become a promising damage detection tool [21,22]. The advantage of wavelet analysis is that it breaks down a signal in a series of functions (wavelets) and allows the identification of local features from the scale and position of wavelets. The signal for example can be the displacement over a region of interest for a structure. Using the wavelet transform, the local features in a spatially distributed structural response signal can be identified with a desired resolution.

Liew and Wang [23] introduced the application of wavelet theory for crack identification in structures. They considered a simply supported beam containing a transverse crack and used a finite difference scheme to calculate the deflection of the beam. In order to determine the location of the crack by wavelet transforming the deflection data they had to use an initial displacement to excite the beam. Deng and Wang [24] applied directly the discrete wavelet transform to structural response signals to locate a crack along the length of a beam. Quek et al. [25] used also wavelet analysis for crack identification in structures. The authors were able to accurately detect small cracks in beams under both simply supported and fixed–fixed boundary conditions. In all the above-mentioned work, however, no attempt was made to estimate the size of the crack. Hong et al. [26] used the Lipschitz exponent for the detection of singularities in beam modal data. The Mexican hat wavelet was used and the damage extent has been related to different values of the exponent. The correlation, however, of the damage extent to the Lipschitz exponent is sensitive to both sampling distance and noise resulting in limited accuracy of the prediction. Wang and Deng [27] extended the use of wavelet analysis to a cracked plate with a through-thickness crack. The investigation, however, was limited to crack localization.

In the present work, a method for crack identification in plate structures based on wavelet analysis is presented. The vibration modes of a plate having an all-over part-through crack parallel to one edge are wavelet transformed and both the location and depth of the crack are estimated. For this purpose, a "symmetrical 4" wavelet having two vanishing moments is utilized. The position of the crack is estimated by the variation of the spatial response signal along a line vertical to the crack due to the high resolution property of the wavelet transform. To estimate the depth of the crack an intensity factor is defined which relates the depth of the crack to the coefficients of the wavelet transform. An intensity factor law is established which allows accurate prediction of the crack depth. The feasibility of the proposed approach is demonstrated through simulation examples which involve plates with all-over part-through cracks of varying depths at different locations. In view of the obtained results, the advantages and limitations of the proposed method as well as suggestions for future work are presented and discussed.

2. Wavelet transform

In this section a brief introduction of the relevant wavelet theory is presented. A more detailed analysis can be found in Refs. [28,29]. Subsequently, the practical way of how the wavelet transform will be used in the present work for crack identification in plates is described.

2.1. Fundamentals

A wavelet is a function with two important properties: oscillation and short duration.

A function $\psi(x)$ is a wavelet if and only if its Fourier transform $\Psi(\omega)$ satisfies

$$\int_{-\infty}^{+\infty} \frac{|\Psi(\omega)|^2}{|\omega|^2}\,\mathrm{d}\omega < +\infty \tag{1}$$

This condition implies that

$$\int_{-\infty}^{+\infty} \psi(u)\,\mathrm{d}u = 0, \tag{2}$$

meaning that a wavelet is an oscillating function with zero mean value. For practical purposes it is also required the wavelet to be concentrated in a limited interval $[-K, K]$, or in other words have compact support. Although wavelets are usually used to analyze signals in the time domain, spatially distributed signals can be equally analyzed by wavelets. The continuous wavelet transform of a function $f(x)$, is defined as

$$Wf(u,s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(x)\psi^*\left(\frac{x-u}{s}\right) \mathrm{d}x, \tag{3}$$

where $\psi^*(x)$ is the complex conjugate of the wavelet function. In translating Eq. (3) one might recognize the inner product of $f(x)$ with scaled and translated versions of the original wavelet function. Large values of scale s correspond to big wavelets and thus coarse features of $f(x)$, while low values of s correspond to small wavelets and fine details of $f(x)$. This inner product is carried out for all times so that the one-dimensional function $f(x)$ is transformed into a two-parameter function $Wf(u,s)$, so that useful information about the function analyzed will be revealed.

An important property of the wavelet transform is its ability to react to subtle changes of the signal structure. To point this out, suppose that the wavelet used is the derivative of a continuous function $\phi(x)$ usually called the scaling function, i.e., $\psi(x) = \mathrm{d}\phi(x)/\mathrm{d}x$. The wavelet transform can be written as

$$Wf(u,s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(x) \frac{\mathrm{d}}{\mathrm{d}u}\phi^*\left(\frac{x-u}{s}\right) \mathrm{d}x = \frac{\mathrm{d}}{\mathrm{d}u}\left\{ \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(x)\phi^*\left(\frac{x-u}{s}\right) \mathrm{d}x \right\}. \tag{4}$$

The continuous wavelet transform is proportional to the first derivative of $f(x)$ smoothed by the function $\phi(x)$. In an analogous way, wavelets that are higher derivatives of a smoothing function can be constructed. The wavelet transform coefficients will be proportional to a smooth version of the second, third or higher derivatives of the signal, respectively. This means that it is possible to examine different rates of change of the signal for all scales of interest allowing a completely local or less local feature extraction procedure.

Of particular importance are the local maxima of $|Wf(u,s)|$ which, as explained above, are the local maxima of the derivative of $f(x)$ smoothed by $\phi(x)$. Mallat and Hwang [30] connected the regularity of a function at a point $x = x_o$ with the decay of the local maxima of the wavelet modulus across scales. To detect singularities one has to examine the asymptotic decay of wavelet modulus maxima in a cone $|x - x_o| < \varepsilon$, where $\varepsilon > 0$ is arbitrarily small, as s tends to zero. If the coefficients decay at some rate as the scale decreases to zero, then x_o is a singular point of $f(x)$.

Traditionally, the regularity of a function at a point of time or space has been characterized by its Hoelder exponent. More specifically, for an isolated singularity, i.e., non-oscillating singularity, the wavelet transform modulus maxima satisfy

$$|Wf(u,s)| \leqslant As^{h+1/2}, \tag{5}$$

where A is a constant and h is the Hoelder exponent. The Hoelder exponent gives information about the differentiability of a function. For example, if the value of the exponent is 1.5 at a point x_o, the function $f(x)$ is one time differentiable but not two times differentiable. The greater the value of the Hoelder exponent, the more regular is the function at this point. If an exponent h is assigned to an isolated singularity at point x_o then it is possible to interpolate all points of the function close to x_o with a curve of the form x^h. A practical way to calculate the Hoelder exponent is obviously by rewriting Eq. (5) in the form

$$\log_2(|Wf(u,s)|) \leqslant \log_2(A) + (h + 1/2)\log_2(s). \tag{6}$$

By keeping the equality sign and plotting the coefficients on a logarithmic scale, A and h are calculated so as the error is minimized in the least-squares sense. Eq. (6) forms the basis of the proposed method for crack identification in plates.

2.2. Wavelet transform as crack identification tool

In order to apply the wavelet transform as a crack identification tool, it is important to clarify the role of the parameters A and h in Eq. (6). The Hoelder exponent h describes the type of a singularity. A Dirac delta function, for example, is Hoelder -1 at $x = 0$. For isolated singularities, one loses one degree of regularity by differentiation and gains one degree of regularity with integration. Thus, a step function discontinuity is described by a Hoelder exponent of $h = 0$, as it is the time derivative of a Dirac delta function. Assume next that all singularities in a signal are of the same type, say for convenience they resemble a step function. In this case, all singularities are characterized by the same exponent. Each singularity, however, might be discriminated from its relative magnitude which is described by the change of constant A. Therefore, constant A can be considered as an intensity factor, i.e., a measure of the severity of a singularity.

To make the described concepts clear, consider a signal with a discontinuity of constant magnitude obeying different laws (Fig. 1). First, the continuous wavelet transform is computed and $\log|Wf(u,s)|$ versus $\log s$ is plotted in Fig. 2. It is obvious that the value of exponent h decreases as the singularity becomes more steep. For example, $h = 0.126$ for the steepest law $x^{0.1}$, while $h = 0.798$ for the law $x^{0.8}$.

Next, the power of x is kept at a constant value 0.5 ($h = 0.507$) and the magnitude of the singularity is varied from 0.01% to 0.2% (Fig. 3). The corresponding wavelet coefficients versus scale are shown in Fig. 4. The wavelet coefficients are parallel lines (constant h) and only the value

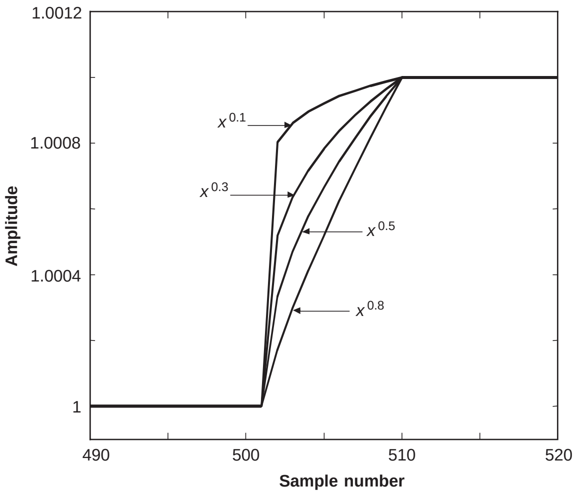

Fig. 1. Functions with an isolated singularity of constant magnitude obeying different laws.

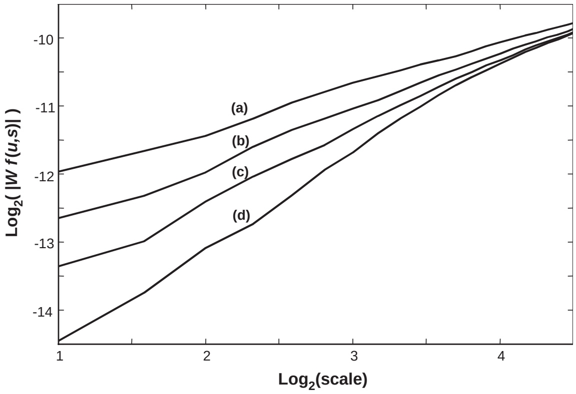

Fig. 2. Wavelet maxima versus scale of the functions shown in Fig. 1. (a) $x^{0.1}, h = 0.126$; (b) $x^{0.3}, h = 0.314$; (c) $x^{0.5}, h = 0.507$; (d) $x^{0.8}, h = 0.798$.

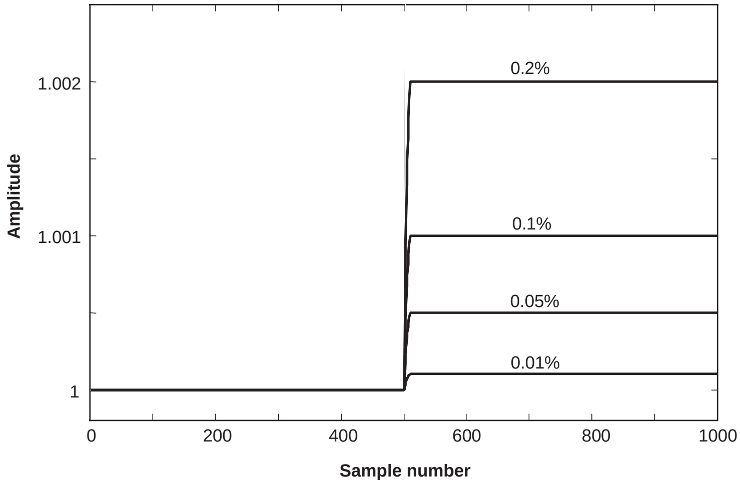

Fig. 3. Functions with an isolated singularity of increasing magnitude obeying the law $x^{0.5}$.

of A changes. The value of constant A increases with increasing singularity magnitude. Therefore, constant A can be considered as an intensity factor relating the wavelet coefficients to the magnitude of the singularity. In a practical application of Eq. (6) for crack identification, the slope h and constant A should be determined by wavelet transforming the response signal of the structure and the size of the damage can be estimated via constant A provided that a link of A to crack size has been established. This procedure will be illustrated in a subsequent section with the help of numerical examples.

Finally, the choice of the analysing wavelet is left to be discussed. In practice, wavelets of higher number of vanishing moments give higher coefficients and more stable performance. On the other hand, one should bear in mind that the effective support of a wavelet increases with the number of vanishing moments. Therefore, a compromise between the number of vanishing moments and adequate localization should be accomplished. Good candidates can be considered the

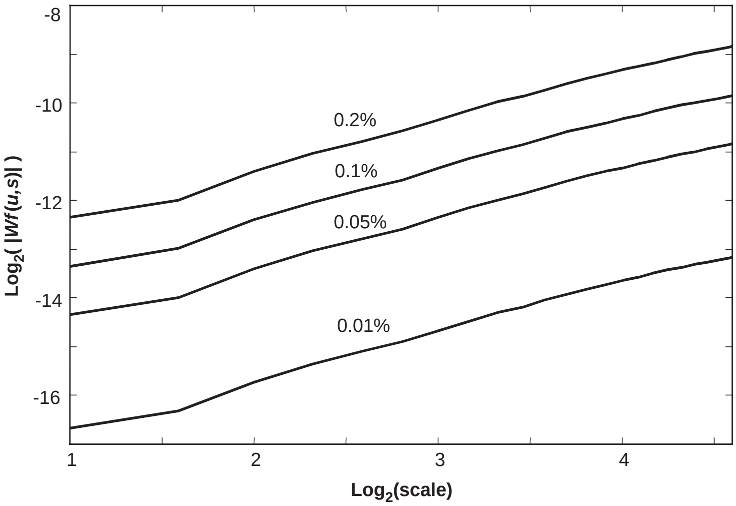

Fig. 4. Wavelet maxima versus scale of the functions shown in Fig. 3. Constant A increases with increasing singularity amplitude. In all cases $h = 0.507$.

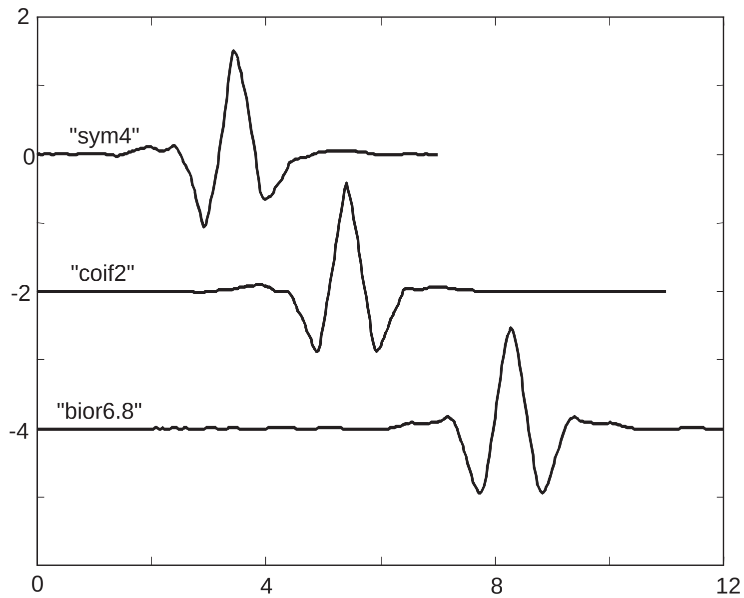

Fig. 5. Examples of the symmetrical, coiflet and biorthogonal family of wavelets.

"symmetrical 4" wavelet with 4 vanishing moments and a support length of 7, the "coiflet 2" wavelet with 4 moments and a support length of 11 and the "biorthogonal 6.8" with 5 moments and a support length of 13 (see Fig. 5). After some experimentation the "symmetrical 4" wavelet has been chosen and used as analysing wavelet throughout the present work.

3. Vibration model of a cracked plate

An elastic rectangular plate of size $a \times b$ and thickness H with an all-over part-through crack running parallel to one side of the plate is considered, as shown in Fig. 6. The crack has a uniform depth ℓ and is considered to be open. A hypothetical boundary along the crack divides the plate into two segments (1) and (2). This model has been analyzed in detail in Ref. [19]. Here, only the basic lines of the analysis are briefly presented for clarity. Considering only bending vibrations, the flexibility of the crack is modelled by calculating the slope discontinuity along the crack as

$$\theta = (12(1 - v^2)/E)\sigma_b \alpha_{bb}, \tag{7}$$

where E is Young's modulus, v is the Poisson's, ratio and σ_b is the nominal bending stress due to bending moments. The compliance coefficient α_{bb}, characterizing the crack is given by the following equation:

$$\alpha_{bb} = \frac{1}{H} \int_0^{\ell} g_b^2 \, \mathrm{d}\ell, \tag{8}$$

where g_b is a dimensionless function of the relative crack depth $\xi = \ell/H$ defined as

$$g_b = \xi^{1/2}(1.99 - 2.47\xi + 12.97\xi^2 - 23.11\xi^3 + 24.80\xi^4). \tag{9}$$

The nominal stress σ_b, expressed in terms of the lateral deflection w, takes the form

$$\sigma_b = \frac{-EH}{2(1 - v^2)} \left(\frac{\partial^2 w}{\partial y^2} + v \frac{\partial^2 w}{\partial x^2} \right). \tag{10}$$

By substituting Eq. (10) into Eq. (7) and using dimensionless space variables $\zeta = x/a$ and $\eta = y/b$ the slope discontinuity at both sides along the hypothetical boundary at $\eta = \eta_0$ is expressed as

$$\theta|_{\eta=\eta_0} = \frac{-6H}{b} \left(\frac{\partial^2 w}{\partial \eta^2} + v\phi^2 \frac{\partial^2 w}{\partial \zeta^2} \right) \alpha_{bb}|_{\eta=\eta_0}, \tag{11}$$

where $\phi = b/a$ is the plate aspect ratio.

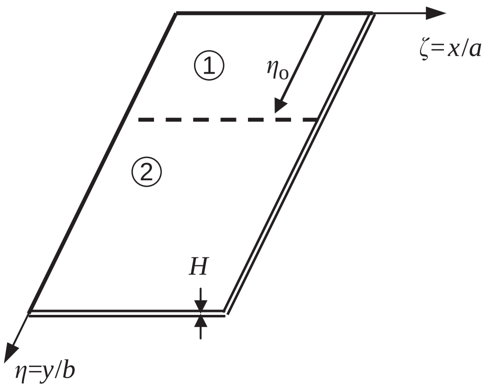

Fig. 6. Rectangular plate with an all-over part-through crack in non-dimensional coordinates.

The governing equation of the free vibration of a plate is

$$-D\nabla^4 w = M(\partial^2 w/\partial t^2), \tag{12}$$

where $w(x, y, t)$ is the transverse deflection, ∇^4 is the biharmonic operator, M is the mass per unit area and $D = EH^3/12(1 - v^2)$ is the flexural rigidity.

Letting

$$w(x, y, t) = W(x, y)\exp(\imath\omega t). \tag{13}$$

the time dependence can be eliminated so as to obtain

$$\nabla^4 W - (M\omega^2/D_E)W = 0. \tag{14}$$

Utilizing Levy's method for a plate simply supported at the two opposite sides $x = 0$ and a (i.e., $\zeta = 0$ and 1), the shape function in terms of the dimensionless space variables (ζ, η) takes the form

$$W(\zeta, \eta) = \sum_{m=1}^{\infty=1} Y_m(\eta)\sin(m\pi\zeta). \tag{15}$$

Substitution of Eq. (15) into Eq. (14) leads to an ordinary differential equation for Y_m, which has as its general solution

$$Y_m(\eta) = A_m \cosh(\beta_m\eta) + B_m \sinh(\beta_m\eta) + C_m \sin(\gamma_m\eta) + D_m \cos(\gamma_m\eta), \tag{16}$$

with

$$\beta_m = \phi\sqrt{\lambda^2 + (m\pi)^2}, \quad \lambda_m = \phi\sqrt{\lambda^2 - (m\pi)^2}, \quad \lambda^2 = \omega a^2\sqrt{M/D}. \tag{17}$$

Here ω is the natural circular frequency of the cracked plate and $A_m - D_m$ are arbitrary constants. The form of Eq. (16) holds for the range of $\lambda^2 > (m\pi)^2$ however, a similar solution can be obtained for the case $\lambda^2 < (m\pi)^2$.

The cracked plate is considered to be simply supported at all four edges. Because of the form of solutions the boundary conditions at $\zeta = 0$ and 1 are automatically satisfied. The other boundary conditions to be considered are applied at the edges $\eta = 0$ and 1 of the two regions separated by the crack line and to the inner boundary $\eta = \eta_0$ along the crack. The mode shape functions in the η direction for regions (1) and (2) when $\lambda^2 > (m\pi)^2$ have the form

$$Y_{1m}(\eta) = A_{1m}\cosh\beta_m\eta + B_{1m}\sinh\beta_m\eta + C_{1m}\sin\gamma_m\eta + D_{1m}\cos\gamma_m\eta.$$

$$Y_{2m}(\eta) = A_{2m}\cosh\beta_m\eta + B_{2m}\sinh\beta_m\eta + C_{2m}\sin\gamma_m\eta + D_{2m}\cos\gamma_m\eta. \tag{18}$$

The boundary conditions at $\eta = 0$ and $\eta = 1$ can be expressed as

$$Y_{1m}(\eta)|_{\eta=0} = \left.\frac{\partial^2 Y_{1m}(\eta)}{\partial\eta^2}\right|_{\eta=0} = 0, \quad Y_{2m}(\eta)|_{\eta=1} = \left.\frac{\partial^2 Y_{2m}(\eta)}{\partial\eta^2}\right|_{\eta=1} = 0. \tag{19}$$

The application of the boundary conditions (19) to Eq. (18) yields the shape functions for the two regions as

$$W_{1m}(\zeta, \eta) = \{B_{1m}\sinh\beta_m\eta + C_{1m}\sin\gamma_m\eta\}\sin(m\pi\zeta), \quad 0 \leqslant \eta \leqslant \eta_0,$$
$$W_{2m}(\zeta, \eta) = \{B_{2m}\sinh\beta_m(\eta - 1) + C_{2m}\sin\gamma_m(\eta - 1)\}\sin(m\pi\zeta), \quad \eta_0 \leqslant \eta \leqslant 1. \tag{20}$$

The inner boundary conditions at the crack location $\eta = \eta_0$ are

$$W_1 = W_2 \,|_{\eta=\eta_0}, \quad M_1 = M_2 \,|_{\eta=\eta_0}, \quad V_1 = V_2 \,|_{\eta=\eta_0}. \tag{21}$$

The above equations express the equality of deflections, bending moments and shear forces at the two sides of the crack location, respectively.

Finally, the slope compatibility condition of the crack is

$$\left.\frac{\partial w_1}{\partial \eta} - \theta - \frac{\partial w_2}{\partial \eta}\right|_{\eta=\eta_0} = 0. \tag{22}$$

By substituting Eqs. (20) into Eqs. (21) and (22) a set of homogeneous equations for the unknown constants B_{1m}, C_{1m}, B_{2m} and C_{2m} is obtained. The vanishing determinant of the coefficient matrix leads to the characteristic equation. Making use of this equation with an integer value of the mode number m, i.e., $m = 1$, the eigenfrequencies, ω_n of the cracked plate are determined, for a specific crack depth ξ and crack location η_0. The lowest eigenfrequency is assigned to $n = 1$. The eigenfunctions of the system determine the vibrational mode shape functions for the two regions (1) and (2), within the arbitrary constant B_{1m} in the form

$$W_1(\zeta, \eta) = B_{1m}\left\{\sinh \beta_m \eta + \frac{e_3 + e_4}{e_5} \sin \gamma_m \eta\right\} \sin(m\pi\zeta).$$
$$W_2(\zeta, \eta) = B_{1m}\left\{e_1 \sinh \beta_m(\eta - 1) + e_2 \frac{e_3 + e_4}{e_5} \sin \gamma_m(\eta - 1)\right\} \sin(m\pi\zeta), \tag{23}$$

where the parameters $e_1 - e_5$ are defined as

$$e_1 = \frac{\sinh \beta_m \eta_0}{\sinh \beta_m(\eta_0 - 1)}, \quad e_2 = \frac{\sin \gamma_m \eta_0}{\sin \gamma_m(\eta_0 - 1)},$$
$$e_3 = 2\beta_m \phi^2 \lambda^2 [\cosh \beta_m \eta - e_1 \cosh \beta_m(\eta_0 - 1)],$$
$$e_4 = \frac{6H}{b} \alpha_{bb} \{\beta_m^2 - \phi^2 \nu (m\pi)^2\}^2 \sinh \beta_m \eta_0,$$
$$e_5 = \frac{6H}{b} \alpha_{bb} \{\beta_m^2 - \phi^2 \nu (m\pi)^2\} \{\gamma_m^2 + \phi^2 \nu (m\pi)^2\} \sin \gamma_m \eta_0. \tag{24}$$

4. Wavelet application for cracked plate analysis

The existence of a crack in a structure results in a decrease in natural frequencies and changes in the mode shapes of vibration. In that vein, simple methods have been developed for crack identification in beam structures via natural frequency changes. In case of cracked plates, however, even moderate cracks have a minor effect on natural frequencies [19,20]. This fact makes a reliable crack identification based on natural frequency changes difficult. Therefore, the application of wavelet analysis as crack detection tool in plate structures becomes important.

4.1. Determination of crack location

In this section the feasibility of the wavelet-based crack identification approach will be investigated using simulated structural response data. For numerical simulations, the cracked plate model described in the previous section is utilized.

The plate is considered to be simply supported at all four edges with an all-over crack at $\eta_0 = 0.2$ running parallel to one of its edges. The following dimensions and mechanical characteristics are considered: $a = 0.2$ m, $b = 0.3$ m, $H = 0.004$ m, $E = 200$ GPa, $\nu = 0.3$, $\rho = 7860\ \mathrm{kg/m^3}$. The depth of the over-all crack is varied from 10% up to 50%.

The first and second modes of vibration are calculated from Eqs. (23) for $m = 1$ and taking $\omega = \omega_1$ and ω_2 respectively. Based on the mode shapes, response data of the plate displacement along vertical lines at different locations are generated. This way, the problem becomes practically one dimensional. A total number of 1001 points are available corresponding to an actual spatial sampling distance of 0.3 mm. Data are normalized so that the maximum displacement value is unity. The displacement response of the cracked plate against the coordinate η along a vertical line at $\zeta = 0.5$ is shown in Fig. 7.

It can be seen that the displacement data reveal no local features that directly indicate the existence of a crack. The structural response data are analyzed using the continuous wavelet transform. The continuous wavelet transform is preferred instead of the discrete version, as the redundancy of information it provides is useful for analysis purposes. The wavelet transform is implemented for scales 1 to 25 with the "symmetrical 4" as the analyzing wavelet. As explained before, one cannot use a scale less than one because of the available resolution. On the other hand, for scales over about 25 the singularity introduced by the crack cannot be considered isolated. This last observation is not important for this study, but is of particular value for a case where more than one crack on a plate has to be accurately localized. If two cracks appear at a distance d,

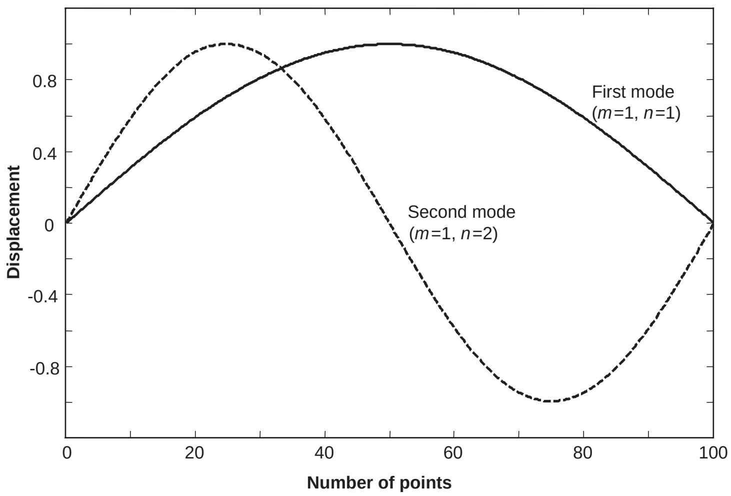

Fig. 7. Calculated displacement response of the cracked plate along a vertical line at $\zeta = 0.5$ for the first and second mode. Crack location $\eta_0 = 0.2$, relative crack depth 20%.

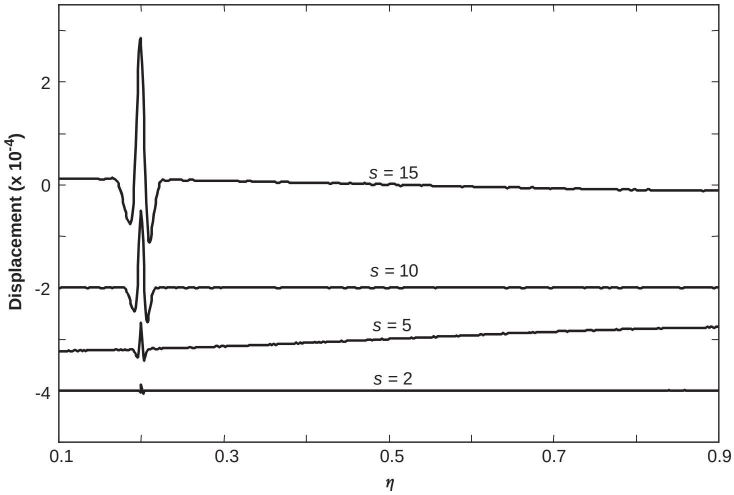

Fig. 8. Wavelet analysis of different scales based on the displacement response of the cracked plate in Fig. 7 (first mode of vibration).

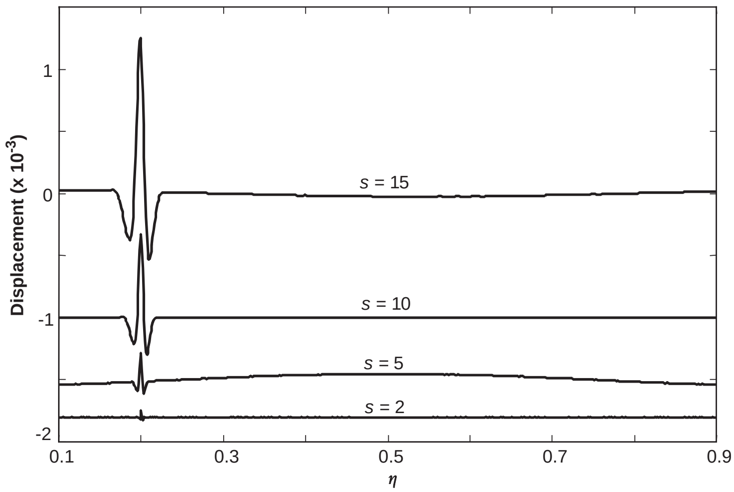

Fig. 9. Wavelet analysis of different scales based on the displacement response of the cracked plate in Fig. 7 (second mode of vibration).

then one should restrict the analysis to scales that the effective support of the wavelet is smaller than the distance d, otherwise the damaged points merge together.

The results of the wavelet analysis are presented in Figs. 8 and 9 for scales 2, 5, 10 and 15. This is the case of a 20% crack and it is obvious that the wavelet transform coefficients exhibit maximum at point $\eta = 0.2$. This implies the presence of a singularity, but to be certain about that one has to observe the trend of wavelet maxima at this point, as the scale decreases. From Figs. 10 and 11 it is clear that the absolute value of the wavelet maxima decreases in a regular manner as the scale decreases. Ideally, it should tend to zero for zero scale, but as mentioned before one is

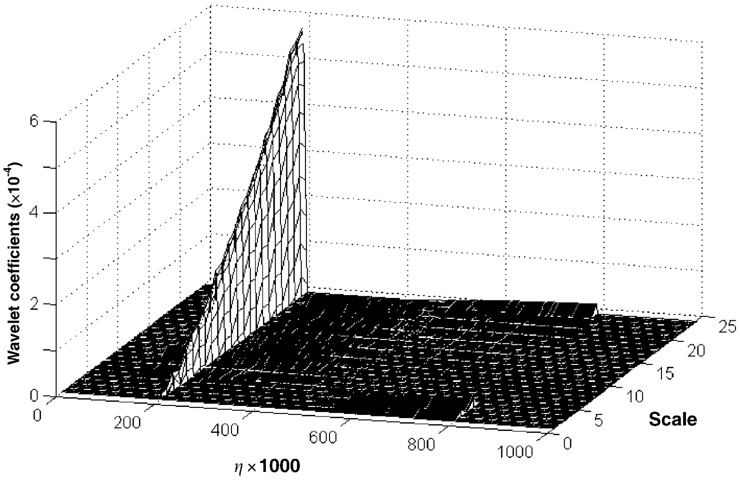

Fig. 10. Three-dimensional plot of the wavelet transform showing the trend of wavelet modulus maxima at crack location (first mode).

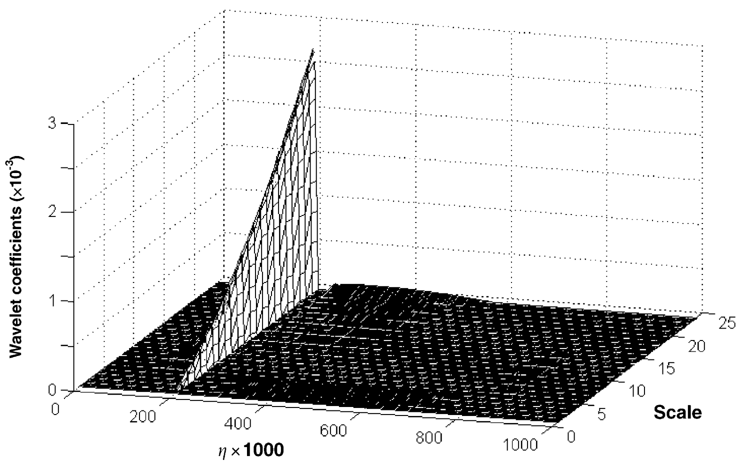

Fig. 11. Three-dimensional plot of the wavelet transform showing the trend of wavelet modulus maxima at crack location (second mode).

forced to draw conclusions from the trend of coefficients to the minimum scale of 1. The second mode of vibration produces higher values for the wavelet coefficients, which means that it is more sensitive for crack detection. All cases investigated presented more or less the same picture with advanced crack depth leading to higher coefficients.

The absence of coefficients of significant value in Figs. 10 and 11 at any other location away from the crack is characteristic. This is attributed to the fact that the analyzed data stem from theoretical computations and hence contain no noise or measurement errors. In a real experiment, however, noise is expected to corrupt the data. It is known [27] that the wavelet transform coefficients behave in a completely different manner when they stem from noise disturbances. It has been shown [30] that random noise is characterized by negative Hoelder exponents and so the wavelet modulus maxima increase as the scale decreases. This observation provides a way to discriminate singular points from noise. The same procedure was repeated in case of displacement response distribution along lines parallel to ζ direction, i.e., parallel to the crack. In this case, the wavelet analysis reveals no sudden changes and hence the crack remains undetected.

In conclusion, the analytical examples demonstrate that a crack induces certain perturbation features in the total structural response along lines perpendicular to the crack. It appears that these local features can be easily revealed by analyzing the spatial response with wavelet transform. Therefore, the location of an existing crack can be easily and accurately determined using wavelet analysis.

4.2. Estimation of crack depth

As already mentioned. Eq. (6) can be used to relate crack size to the coefficients of the wavelet transform via the constant A. For that purpose, one has to examine the modulus maxima lines of the wavelet transform.

For the plate considered in the previous section, Fig. 12 shows a plot of the logarithm of the wavelet coefficients versus scale for response data corresponding to the first mode of vibration. The location of the crack is fixed ($\eta = 0.2$) while its depth is varied from 10% up to 50%.

The wavelet coefficients are parallel lines of constant slope, or in other words of constant exponent h. The Hoelder exponent for all cases has a constant value equal to 1. This means that the mode function is one time differentiable at the location of the crack. The constant value of the Hoelder exponent implies singularities of the same type caused by the same physical cause, which in our case is the existence of a crack. Similar results are obtained from the analysis of response data corresponding to the second mode of vibration (Fig. 13). These results validate to an extent the fact that the Hoelder exponent characterizes the type of the defect, while constant A depends

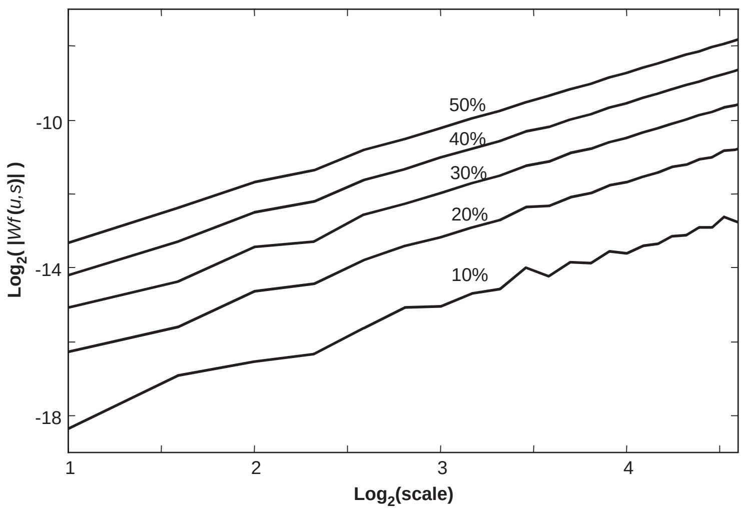

Fig. 12. Wavelet maxima coefficients versus scale for different crack depths (first mode).

on the size of the defect. In both cases the intensity factor A increases with increasing depth. This suggests a possibility to correlate the crack size to the constant A.

In order to use constant A for quantitative crack sizing, a relation between A and crack size needs to be established. For that purpose, the variation of constant A with crack size has been systematically investigated. Using the results shown in Figs. 12 and 13 (wavelet coefficients versus scale for fixed crack location and different crack size) the coefficient A has been evaluated for different crack depths by linear interpolation.

Fig. 14 presents the intensity factor A versus crack depth for the first and second mode of vibration. It can be seen that in both cases the intensity factor increases with increasing crack

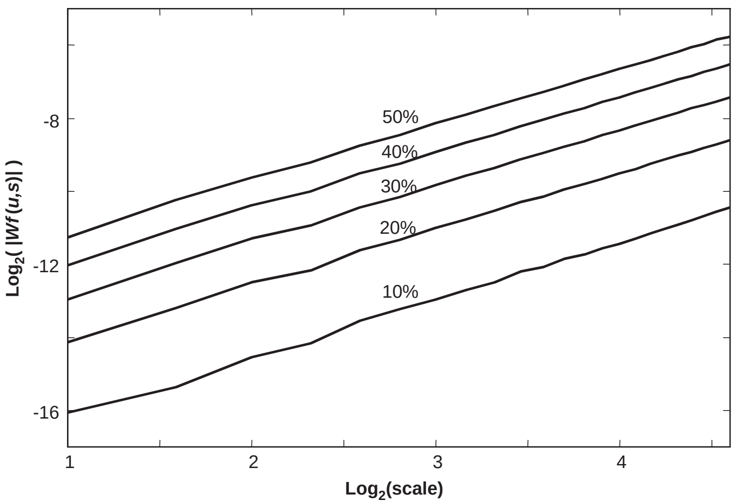

Fig. 13. Wavelet maxima coefficients versus scale for different crack depths (second mode).

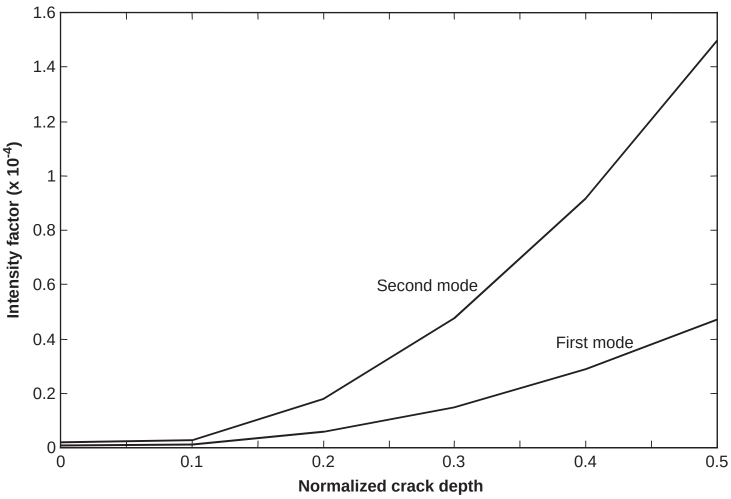

Fig. 14. Intensity factor versus normalized crack depth. Crack location $\eta_0 = 0.2$.

depth following a second order polynomial law. Therefore, Fig. 14 can be used for estimating the relative crack depth for a given intensity factor. For the plate under consideration, the calculated value of the intensity factor $A = 1.9 \times 10^{-5}$ leads to a value of 20% for the relative crack depth in agreement with the assumed model.

5. Conclusions

A method for crack detection in cracked plates based on wavelet analysis has been presented. The viability of the method has been demonstrated by analyzing the vibration modes of a plate with an all-over part-through crack parallel to one edge of the plate using the "symmetrical 4" wavelet.

The location of the crack was determined by the sudden changes in the spatial response of the transformed signal at the site of the crack. Such local changes are not obvious from the response data they are, however, discernible as singularities when using wavelet analysis.

For the estimation of the relative crack depth an intensity factor was established. It relates the size of the crack to the corresponding wavelet coefficients. It was shown that the intensity factor changes with crack depth according to a second order polynomial law and therefore, can be used as an indicator for crack extent.

In conclusion, the presented results provide a foundation for using wavelelet analysis as an efficient crack detection tool in plate structures. The advantage of using wavelet analysis is that local features in a displacement response signal to be analyzed picks up the perturbations caused by the presence of the crack. Further work is needed, however, to advance crack detection in plates using wavelet analysis. A key issue is the spatial resolution and accuracy of the measurements. Therefore, actual response data are needed to demonstrate the practicality of the method. Furthermore, a detailed theoretical correlation of the intensity factor to the physical crack size would enhance the reliability and accuracy of the proposed method. Identification of cracks of finite length in plates is also a problem of practical interest to be investigated. Work is already under way on the above mentioned issues and will be the subject of a future publication.

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